

I'd put my money on the sun and solar energy. What a source of power! I hope we don't have to wait 'til oil and coal run out before we tackle that.

Thomas Edison, Scientist and Inventor

10-1 Utility Expenses

Objectives

- Compute the cost of electric, gas, oil, and water for the home.
- Compute the cost of using specific appliances for specific lengths of time.
- Compute the time it takes an energy-saving appliance to pay for itself.

Common Core

N-Q1, N-Q2, A-SSE1a, A-SSE1b

Key Terms	• utility	• kilowatt-hour (kWh)	• previous reading
	• meter	• cubic foot	• present reading
	• watt	• ccf	
	• watt-hour	• volume	

Tammy draws a scatterplot and regression line plotting her hours worked (x) and tips (y). Which represents the units of the slope?

CCSS Warm-Up

- a. dollars b. hours/dollar c. dollars/hour

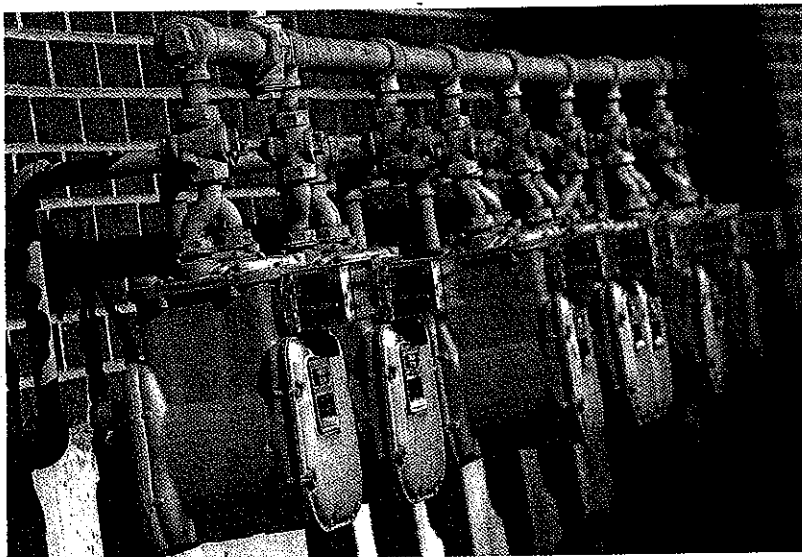
HOW MUCH WILL IT COST TO RUN THE UTILITIES IN YOUR HOME?

If you own a home or rent an apartment, you are charged for using electricity, natural gas, heating oil, and water. These services are **utilities**. You pay for utilities after you use them, so you are actually using credit when you purchase utilities. Many people don't think about the costs of these services as they are using them.

How much electricity do you use? You probably don't know the exact amount, but it is recorded by a **meter**. An employee of the electric company comes to your home to read this meter to determine how much electricity you used. Every electrical appliance uses electricity. Electricity is measured in **watts**. The amount of electricity used is measured in **watt-hours**. For example, a 60-watt light bulb burning for two hours uses 120 watt-hours. A **kilowatt-hour (kWh)** is

equivalent to 1,000 watt-hours of electrical use. Electricity is sold by the kilowatt-hour.

Natural gas and water are sold by the **cubic foot**, which represents the amount of space the gas or water occupies, not the weight. The unit **ccf** represents 100 cubic feet. The amount of space is the **volume**. You can think of one cubic foot as the space occupied by a box that measures one foot on each edge. Natural gas usage is also checked by someone reading a meter in your home. Most communities use water meters to monitor your water consumption.



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Skills and Strategies

Here you will learn how your use of natural gas, water, and electricity is measured and how to read meters and utility bills. Electric and gas meters have dials that display usage. The units for natural gas are 100 cubic feet (ccf) and the units for electric usage are in kilowatt-hours (kWh). A **previous reading** is the last time the meter was read. A **present reading** is the current meter reading.

EXAMPLE 1

Tom's October water bill listed two meter readings. The previous reading was 3,128 ccf and the present reading is 3,141 ccf. How much water did Tom's household use during the billing period?

SOLUTION Subtract to find the number of ccf of water used.

$$3,141 - 3,128 = 13$$

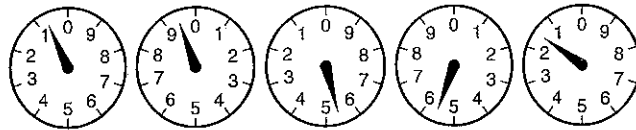
Tom's household used 13 ccf of water during the billing period.

■ CHECK YOUR UNDERSTANDING

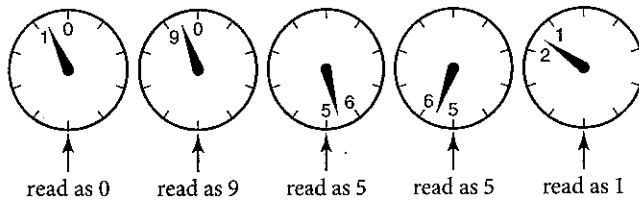
Ron used x ccf of water during a summer month. Express the number of cubic feet of water he used algebraically.

EXAMPLE 2

Janet works for a utility company and is reading the Saevitz household's electric meter. What is the reading?



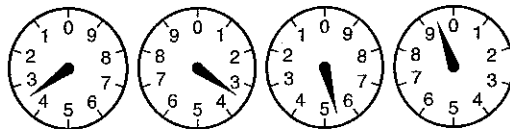
SOLUTION Notice where the arrow points on each dial. The correct reading is the lower of the two numbers closest to the arrow, with one exception. If the arrow points between 0 and 9, this is read as 9.



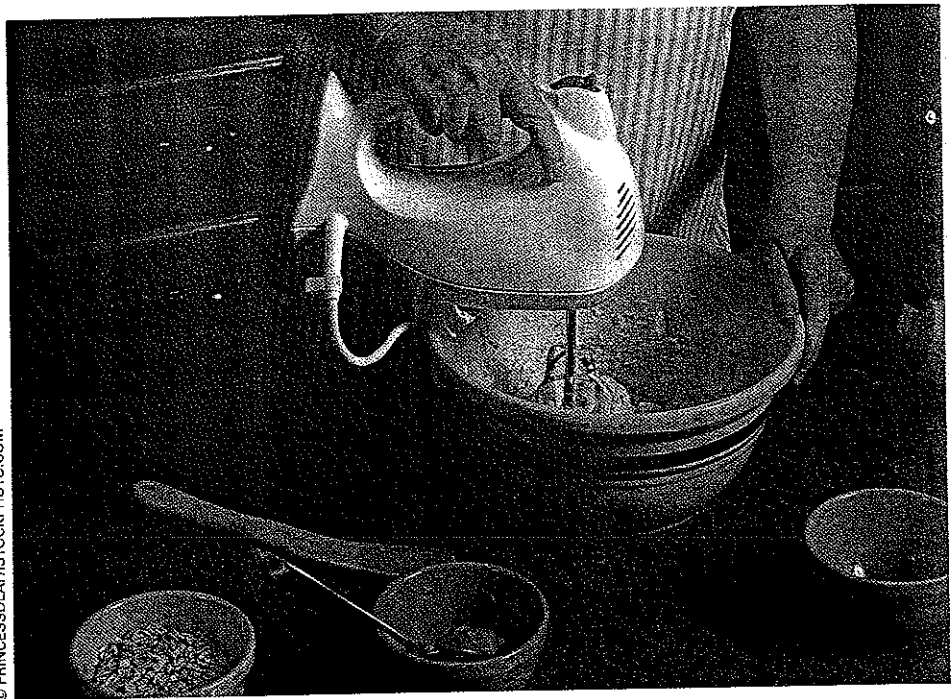
This meter shows a reading of 9,551 kWh of electricity. Some utility companies are installing digital meters which can be read more easily.

■ CHECK YOUR UNDERSTANDING

What is the ccf reading indicated by the dials?



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EXAMPLE 3

A certain electric mixer requires 125 watts. How much would it cost to run the mixer for a total of 90 minutes at a cost of \$0.10 per kilowatt-hour?

SOLUTION Calculate the operating cost of an electrical appliance by checking its wattage and the time it is being used.

Convert minutes to hours. $90 \text{ minutes} = \frac{90}{60} \text{ hour} = 1.5 \text{ hours}$

The number of watt-hours is the product of the watts and the number of hours the appliance is in use.

Find watt-hours. $125 \times 1.5 = 187.5 \text{ watt-hours}$

Electric usage is billed by the kilowatt-hour, which is 1,000 watt-hours. To express the 187.5 watt-hours as kilowatt-hours, divide by 1,000.

Find kilowatt-hours. $187.5 \text{ watt-hours} = \frac{187.5}{1,000} = 0.1875 \text{ kilowatt-hours}$

If a kilowatt-hour costs \$0.10, multiply the kilowatt-hours by \$0.10 to find the cost of using the mixer for 90 minutes.

Find cost per kilowatt-hour. $0.1875 \times 0.10 = 0.01875$

The mixer costs \$0.01875 to run for 90 minutes, or 1.9¢. This is between 1 and 2 cents.

■ CHECK YOUR UNDERSTANDING

An appliance uses w watts. If you run it for m minutes, and the cost per kilowatt-hour is c , express the cost of running the appliance for m minutes algebraically.

EXAMPLE 4

Depending on the climate, utility usage can vary according to the seasons. Electric bills can be higher in the summer due to air conditioning use, and gas and oil usage can be higher in winter due to heating needs. For this reason, some utility companies offer balanced billing. This evens out the monthly cost of utilities to make the homeowner's payments predictable. Last year, the Ross family spent \$3,336 for electricity. They are opting to use balanced billing for next year. What will their monthly payment be?

SOLUTION Balanced billing uses the previous year to determine monthly payments for the upcoming year. Divide the annual cost by 12 to get the monthly cost.

$$\frac{\text{annual utility cost}}{12 \text{ months per year}} = \frac{3,336}{12} = 278$$

The Ross' monthly electric bill will be \$278 under the balanced billing program.

■ CHECK YOUR UNDERSTANDING

Two years ago, the Halloran family used y dollars of electricity. Last year, they used balanced billing. During last year, they used x dollars of electricity, and their balanced billing payments were not enough to pay for their electric usage. They had to pay the difference at the end of the year. Express algebraically the amount they owed the utility company.

EXAMPLE 5

Energy savings often pay for themselves. This means that the savings in energy usage can equal, or offset, the cost of the item after a certain number of years. The Thomson's old water heater costs them \$455 per year to run. The new one they purchased for \$1,240 will save them 31% annually in energy costs to run it. In how many years will it pay for itself?

SOLUTION Compute the savings per year by finding 31% of \$455.

$$455 \times 0.31 = 141.05$$

Divide the cost of the new appliance by the annual savings in energy.

$$\frac{1,240}{141.05} \approx 8.791$$

The water heater will pay for itself in about 9 years, since it will take more than 8 years to save more than \$1,240.

■ CHECK YOUR UNDERSTANDING

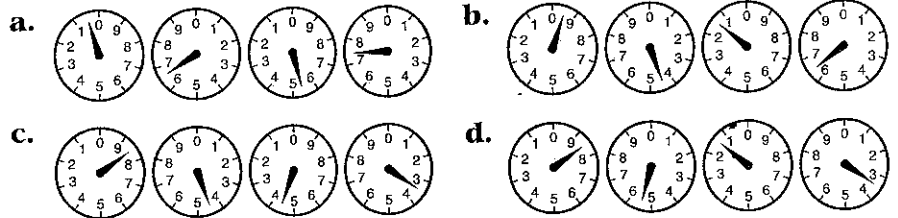
An old dishwasher costs r dollars to run for a year. It is replaced by a new energy-efficient dishwasher that costs c dollars, but saves p percent per year in energy usage. Express algebraically the number of years it will take for the dishwasher to pay for itself.

Applications

I'd put my money on the sun and solar energy. What a source of power! I hope we don't have to wait 'til oil and coal run out before we tackle that.

Thomas Edison, Scientist and Inventor

- Interpret the quote in the context of what you know about the major issues in energy consumption today.
- Emily's last water bill listed a previous reading of 7,123 ccf and a present reading of 7,171 ccf. Her water company charges \$0.73 per ccf of water. What should Emily have been charged on her last water bill?
- What is the meter reading, in ccf, indicated by each of the gas meters shown?



- Bill Heckle's last electric bill is shown below.

WATTCO LIGHTING CORPORATION						
FOR SERVICES TO:	Bill Heckle 12 Cavern St. Linwood, KS 66052			ACCOUNT NUMBER		8761-21
				BILL DATE		AUG 11
SERVICE	SERVICE PERIOD		METER READING		USAGE	AMOUNT
	FROM	TO	PREVIOUS	PRESENT		
ELECTRIC	JUL 3	AUG 4	21,780	24,100	2,320	\$255.20

- What was the previous reading?
 - What is the present reading?
 - How many kilowatt-hours of electricity did Bill use during the service period shown?
 - What did Wattco charge per kilowatt-hour of electricity?
 - The bill covers 31 days of electric use. What was Bill's average daily expense for electricity for this service period? Round to the nearest cent.
- Home heating oil is sold by the gallon. Last winter, the Romano family used 370 gallons of oil at a price of \$3.91 per gallon. If the price increases 9% next year, what will their approximate heating expense be? Round to the nearest ten dollars.
 - The PA system at North High School requires 400 watts when it is switched on. How much would it cost to run for 3 hours, at a cost of \$0.10 per kilowatt-hour?

7. The Zwerling family installed central air conditioning in their house this summer. They are comparing the electric bills of this summer and last summer. The data is shown.

Month	This Summer	Last Summer
June	\$311.20	\$179.90
July	300.65	\$203.40
August	302.50	\$201.11

- What was the total electric bill this summer?
 - What was the total electric bill last summer?
 - Did the bill increase more or less than 50%?
8. Last winter, Anne was charged \$838.35 for 9,315 kWh of electricity. What did her company charge per kilowatt-hour of electricity?
9. A certain appliance requires 225 watts when it is switched on. How much would it cost to run for m minutes, at a cost of d dollars per kilowatt-hour? Express your answer algebraically.
10. Jessica's parents are always telling her to turn off the lights when she leaves a room. A light bulb requires 75 watts to run when it is turned on. The fixture in Jessica's room requires four of these bulbs.
- Jessica's parents estimate that she leaves the lights on unnecessarily for 2.5 hours per day. How many watt-hours of electricity are used by these bulbs during 2.5 hours?
 - Approximately how many kilowatt-hours of electricity are used in a year to keep these bulbs lit for 2.5 hours per day?
 - At a cost of \$0.09 per kilowatt-hour, how much money is wasted per year by keeping these lights on unnecessarily? Round to the nearest dollar.
 - If five million teenagers keep lights on as Jessica does, how much is wasted in unnecessary electric expenses?
11. The Smithtown Water Company uses water meters that measure water usage in gallons. They charge \$0.12 per gallon of water. If Jack's previous meter reading was 45,621 gallons and his present water reading is 46,555 gallons, what is the amount of his water bill?

12. Ron Sargeant's electric bill from the Longwood Power Authority is shown. If his meter reading for December 17 is 52,344, find the total charges for the December bill. Include all rates and charges as shown on the November bill.

LONGWOOD POWER AUTHORITY	
ELECTRIC USAGE	DELIVERY AND SYSTEM CHARGES
<u>METER READING</u>	Basic Services
NOVEMBER 17 50,361	31 days @ .18 \$ 5.58
OCTOBER 17 49,356	1,005 kWh @ .095 .. 95.48
kWh USED IN 31 DAYS 1,005	Subtotal 101.06
	Sales Tax (3%) 3.04
	Total charges \$104.10

13. Last year, the Forrester family spent \$1,882.56 for electricity. They are opting to use balanced billing for next year.
- What will their monthly payment be under balanced billing?
 - Last year, they had their highest bill in the summer, for \$405.67. Their lowest bill was in the winter. Explain why their lowest bill could not be \$178.

14. The Zlotnick family pays \$223 per month for electricity under balanced billing. At the end of the year, they had used more electricity than the balanced billing covered, and they owed the utility company x dollars. Express their total electricity expenses for the year algebraically.
15. The Waldner family paid their electric bill using balanced billing all last year. The monthly payment was m dollars. At the end of the year, the electric company told them they had a credit of c dollars due to overpayment. This meant they paid for more electricity than they used. Express the value of the electricity used by the Waldners last year algebraically.
16. A certain appliance uses w watts to run. If you run it for m minutes, and the cost per kilowatt-hour is c , the cost of running the appliance for m minutes is given by the formula

$$\frac{w\left(\frac{m}{60}\right)}{1,000}(c)$$

Find the cost of running an appliance that requires 500 watts for 25 minutes at a cost of \$0.125 per kWh. Round to the nearest cent.

17. A large appliance such as a water heater is only running when it has clicked on and is actually heating water. The time your water heater is on varies according to how much you do laundry, take showers, or run the dishwasher. The national average is 3 hours per day.
- If a water heater uses 4,200 watts, find the daily cost of running it at a cost of \$0.11 per kilowatt-hour. Round to the nearest ten cents.
 - Find the annual cost of running the water heater to the nearest ten dollars.
 - A certain energy-saving water heater sells for \$1,100. It will save 36% in energy costs per year compared to the water heater from parts a and b. What will be the approximate annual cost of running this water heater? Round to the nearest ten dollars.
 - In how many years will the new water heater "pay for itself"?
18. In September 1997, the average cost for 1 gallon of home heating oil in New York City was \$1.192 per gallon. By September 2008, it had risen to \$4.173 per gallon. What was the percent increase in those 11 years? Round to the nearest percent.
19. Air conditioners are rated by BTUs. You learned about BTUs in Lesson 8-2, Example 6. One watt-hour is equivalent to 3.413 BTUs per hour.
- How many BTUs per hour are equivalent to a kilowatt-hour?
 - If an air conditioner is rated at 16,000 BTUs, how many watts does it require per hour? Round to the nearest 100 watts.
 - Express your answer to part b in kilowatt-hours.
 - If you run an air conditioner for 18 hours per day for 31 days in July, how many kilowatt-hours will it require for the month? Round to the nearest 100 kilowatt-hours.
 - Based on your answer to part d, and a rate of \$0.13 per kilowatt-hour, estimate the cost of running the air conditioner for July to the nearest 10 dollars.

Globalization, as defined by rich people like us, is a very nice thing . . . you are talking about the Internet; you are talking about cell phones; you are talking about computers. This doesn't affect two-thirds of the people of the world.

Jimmy Carter, 39th President of the United States, in 2002

Electronic Utilities

10-2

Key Terms • electronic utilities

Find the two closest integers to each real number.

1. $\sqrt{17}$

2. $\sqrt{80}$

CCSS Warm-Up

3. $\sqrt{65}$

Objectives

- Compute the cost of cell phone calls, text messaging, Internet service, and cable television.
- Compare different plans for these services.

Common Core

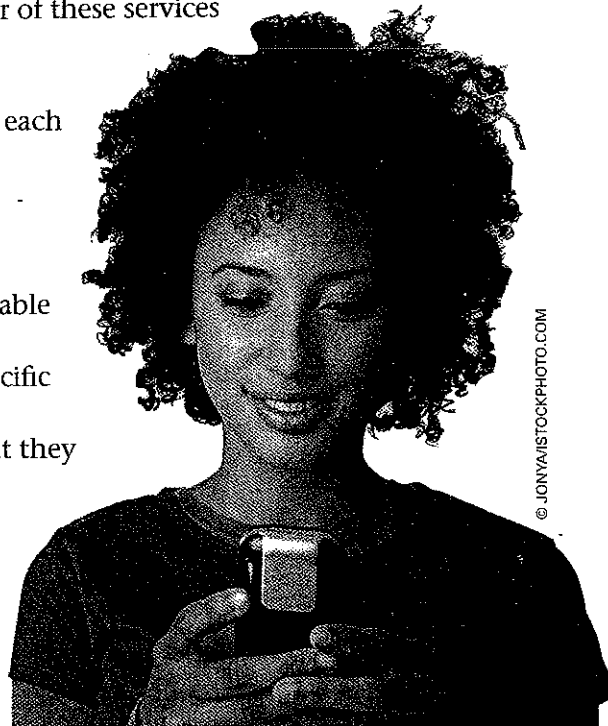
A-SSE1a, F-IF7a, F-IF7b

HOW MUCH DO ELECTRONIC UTILITIES COST TO USE?

When people first landed on the moon in the summer of 1969, cell phones, graphing calculators, personal home computers, and the Internet were more likely to appear in a science fiction movie than anywhere else! Cable television, computers, and digital electronics were in their infancy. Hand-held calculators were bulky, expensive, and rare and had very simple features. The digital wristwatch was new. The explosion of technological advancements has affected our lives tremendously. Imagine life without **electronic utilities** such as Internet access, cell phones, and television. Including these items as a part of daily life has a big affect on a household budget. Whether or not you pay your own bills, as a user of these services you need to know about the costs involved.

- Do you spend much time on the Internet?
- How many text messages do you send and receive each month?
- Do you know the provisions of your cell phone contract?
- Do you watch cable or satellite television?
- What do you know about the different plans available to users of these services?
- Are you sure you have the optimal plan for your specific usage?
- Are you checking your bills each month to see that they are correct?
- Could you change or adjust your usage to lower your costs?

In a year, a household could spend thousands of dollars on electronic utilities. The same household could save hundreds of dollars by using cell phone family plans, combining Internet, cable TV, and phone providers, or switching companies or plans.



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Skills and Strategies

The next examples will help you understand billing procedures and costs of different electronic utilities. Policies vary from company to company and from state to state. Remember that plan offerings change frequently.

EXAMPLE 1

Even with cell phones, there are still hundreds of thousands of pay phones across the United States. A pay phone at a local airport charges \$0.35 for the first three minutes (or part of) and \$0.19 for each extra minute (or part of). You get charged for an entire minute regardless of the portion of that minute you used. Find the cost of a $7\frac{1}{2}$ -minute phone call.

SOLUTION The first three minutes cost \$0.35. Subtract to find the number of minutes over the 3 minutes.

$$7\frac{1}{2} - 3 = 4\frac{1}{2}$$

The call went $4\frac{1}{2}$ minutes over the initial 3 minutes. Therefore, you must pay for an additional 5 minutes.

Multiply to find the cost of the extra 5 minutes.

$$5 \times 0.19 = 0.95$$

The extra 5 minutes cost \$0.95.

Add to find the total cost of the call.

$$0.35 + 0.95 = 1.30$$

The total cost of the call is \$1.30.

■ CHECK YOUR UNDERSTANDING

Find the cost of a $12\frac{1}{4}$ -minute phone call using the prices above.

EXAMPLE 2

Use the greatest integer function to express the payment from Example 1 as a piecewise (split) function.

SOLUTION Let m represent the length of the call in minutes. Let $c(x)$ represent the cost of the call.

First, represent the cost of a call that is 3 minutes or less, which is 35 cents.

$$c(m) = \begin{cases} 0.35 & \text{when } m \leq 3 \end{cases}$$

Next, represent the cost of a call longer than 3 minutes, when m is an integer. The first 3 minutes cost \$0.35, but you must add on a charge of \$0.19 for each minute over the three minutes.

Therefore, use $m - 3$ to represent the number of minutes over 3.

$$c(m) = \begin{cases} 0.35 & \text{when } m \leq 3 \\ 0.35 + 0.19(m - 3) & \text{when } m > 3 \text{ and } m \text{ is an integer} \end{cases}$$

If m is greater than 3 but is not an integer, use the *greatest integer function*. If you use a portion of a minute, you must pay for the entire minute.

If m is greater than 3, and m is not an integer, you must pay for

$$[m - 3] + 1 \text{ minutes}$$

You need to add 1 minute because the greatest integer function removes the portion of your last minute.

Add this line to the piecewise function.

$$c(m) = \begin{cases} 0.35 & \text{when } m \leq 3 \\ 0.35 + 0.19(m - 3) & \text{when } m > 3 \text{ and } m \text{ is an integer} \\ 0.35 + 0.19([m - 3] + 1) & \text{when } m > 3 \text{ and } m \text{ is not an integer} \end{cases}$$

This is the completed piecewise function.

If you substitute $7\frac{1}{2}$ for m in the piecewise function, you need to use the third line of the function.

$$0.35 + 0.19\left(\left[7\frac{1}{2} - 3\right] + 1\right) = 0.35 + 0.19(4 + 1) = 1.30$$

Notice your answer is the same: \$1.30.

The key step to remember is adding the extra +1 because the greatest integer function removes the portion of your last minute.

■ CHECK YOUR UNDERSTANDING

A pay phone at a baseball stadium charges \$0.65 for the first five minutes (or part of) and \$0.22 for each extra minute (or part of). Express the cost $c(m)$ of an m -minute phone call as a piecewise function using the greatest integer function.

EXAMPLE 3

The Call-Tech cellular phone company has many different calling plans. The Tell-Cell plan has a basic charge per month, which includes a certain number of free minutes. There is a charge for each additional minute. The piecewise function below gives the price, $f(x)$, of an x -minute phone call. Fractions of a minute are charged as if they were a full minute.

$$f(x) = \begin{cases} 40 & \text{when } x \leq 750 \\ 40 + 0.35(x - 750) & \text{when } x > 750 \text{ and } x \text{ is an integer} \\ 40 + 0.35([x - 750] + 1) & \text{when } x > 750 \text{ and } x \text{ is not an integer} \end{cases}$$

Describe the cost of Call-Tech's Tell-Cell plan by interpreting the piecewise function.

SOLUTION Notice that, regardless of the number of minutes, there is a basic charge of \$40 per month. This is highlighted in yellow below.

$$f(x) = \begin{cases} 40 & \text{when } x \leq 750 \\ 40 + 0.35(x - 750) & \text{when } x > 750 \text{ and } x \text{ is an integer} \\ 40 + 0.35([x - 750] + 1) & \text{when } x > 750 \text{ and } x \text{ is not an integer} \end{cases}$$

Notice that x , the number of minutes, is always being compared to the number 750 as shown highlighted in blue below. The 750 is the number of free minutes.

$$f(x) = \begin{cases} 40 & \text{when } x \leq 750 \\ 40 + 0.35(x - 750) & \text{when } x > 750 \text{ and } x \text{ is an integer} \\ 40 + 0.35([x - 750] + 1) & \text{when } x > 750 \text{ and } x \text{ is not an integer} \end{cases}$$

The expression $x - 750$ gives the number of additional minutes. Notice that each additional minute costs \$0.35, which is highlighted in green below.

$$f(x) = \begin{cases} 40 & \text{when } x \leq 750 \\ 40 + 0.35(x - 750) & \text{when } x > 750 \text{ and } x \text{ is an integer} \\ 40 + 0.35([x - 750] + 1) & \text{when } x > 750 \text{ and } x \text{ is not an integer} \end{cases}$$

The basic charge is \$40 per month with 750 free minutes, with a charge of \$0.35 per minute for minutes over the 750.

■ CHECK YOUR UNDERSTANDING

Find the monthly cost for someone who had the plan in Example 3 who used 1,234 minutes last month.

EXAMPLE 4

Julianne has a cell phone and gets charged for text messages each month. She is thinking of paying a flat fee of \$40 for unlimited text messaging. If the cost is \$0.15 per text message, how much would she save by using the unlimited plan instead of the pay-per-message plan if she sends about 800 messages each month?

SOLUTION Julianne must first compute the cost of 800 texts at the cost of \$0.15 per text by multiplying.

$$0.15 \times 800 = 120$$

The 800 texts would cost her \$120.

Subtract the cost of unlimited texting to find out how much she saves.

$$120 - 40 = 80$$

Julianne would save \$80 by using the unlimited plan.

■ CHECK YOUR UNDERSTANDING

Kristin's phone company charges \$45 for unlimited texting per month, or \$0.12 per text message sent or received. For what amount of text messages would the unlimited plan cost the same as the per-text plan?

EXAMPLE 5

The Optizone cable TV/Internet/phone provider advertises a flat \$95 per month for all three services for a new customer's first year. The company estimates that this will increase 10% for the second year. Pauline normally pays \$49 for her monthly home phone service, \$35 for Internet service, and \$50 for cable television. If Pauline's usage remains the same, how much will she save per month in the second year?

SOLUTION Compute the monthly sum of Pauline's three services.

$$49 + 35 + 50 = 134$$

Pauline regularly pays \$134 per month for these three services.

Increase the \$95 fee by 10%. $95(0.10) + 95 = 9.50 + 95 = 104.50$

After the first year promotion, Pauline will pay \$104.50.

Subtract to find how much she would save. $134 - 104.50 = 29.50$

Pauline would save \$29.50 per month.

■ CHECK YOUR UNDERSTANDING

How much would Pauline save if the second-year increase was 25% instead of 10%?

EXAMPLE 6

The piecewise function below gives the cost $c(x)$ of x text messages from Horizon Cellular. Graph the function.

$$c(x) = \begin{cases} 32 & \text{when } x \leq 400 \\ 32 + 0.11(x - 400) & \text{when } x > 400 \end{cases}$$

SOLUTION Notice there is a flat fee of \$32 if the number of text messages is 400 or less. Because the domain is the number of text messages sent, the values of x must be integers. For integral values of x from 0 to 400, this can be graphed as ordered pairs that lie on a horizontal line

$$c(x) = 32$$

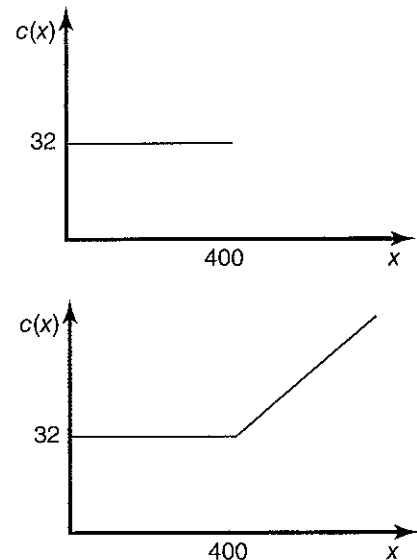
If the number of texts exceeds 400, there is a charge of \$0.11 per text for the texts over 400. There are $x - 400$ texts over 400. Multiply to find the cost of these texts.

$$0.11(x - 400)$$

Add the \$32 fee. $32 + 0.11(x - 400)$

For the integral values of x greater than 400, the ordered pairs lie on the line

$$c(x) = 32 + 0.11(x - 400)$$

**■ CHECK YOUR UNDERSTANDING**

For the graph in Example 6, what are the coordinates of the cusp?

Applications

Globalization, as defined by rich people like us, is a very nice thing . . . you are talking about the Internet; you are talking about cell phones; you are talking about computers. This doesn't affect two-thirds of the people of the world.

Jimmy Carter, 39th President of the United States, in 2002

- Interpret the quote in the context of what you know about the standard of living throughout the world.
- A pay phone at a shopping mall charges \$0.68 for the first four minutes and \$0.21 for each extra minute (or part of a minute).
 - Find the cost of a 10-minute call on this phone.
 - Find the cost of a 13.44-minute call on this phone.
- A phone company set the following rate schedule for an m -minute call from any of its pay phones.

$$c(m) = \begin{cases} 0.70 & \text{when } m \leq 6 \\ 0.70 + 0.24(m - 6) & \text{when } m > 6 \text{ and } m \text{ is an integer} \\ 0.70 + 0.24(\lceil m - 6 \rceil + 1) & \text{when } m > 6 \text{ and } m \text{ is not an integer} \end{cases}$$

- What is the cost of a call that is under six minutes?
 - What is the cost of a 14-minute call?
 - What is the cost of a $9\frac{1}{2}$ -minute call?
- The Tell-All Phone Company charges \$0.58 for the first two minutes and \$0.21 for each extra minute (or part of a minute). Express their rate schedule as a piecewise function. Let m represent the number of minutes and let $c(m)$ represent the cost of the call.
 - Phone companies itemize charges on monthly bills. There are several fees, call surcharges, and taxes. There could be charges for maintenance of the phone wiring within the home. These charges inflate the bill because they are not included in the advertised basic service rate. Be sure to inquire about all extra fees when choosing a phone company. A bill from the Tell-All Phone Company is shown.
 - Find the total amount, a , that must be paid for just the extra charges shown on the bill.
 - Find the total, b , that must be paid for this billing period.
 - What was the percent increase of the total bill when compared to the basic service cost?
Round to the nearest percent.
 - If the basic service cost is b dollars, and the extra charges are e dollars, represent the extra charges as a percent of the basic cost algebraically.

TELL-ALL PHONE COMPANY	
Basic Services	\$39.99
<u>Additional charges</u>	
FCC Line charge	\$6.42
Federal surcharge	0.73
9-11 Surcharge	0.35
Federal tax	0.17
State and Local tax	0.64
Wire Maintenance Option	4.55
Total (Additional charges)	\$ a
<u>Billing Period (May 30 - Jun 30)</u>	
Total charges	\$ b

6. Vicki's phone company charges x dollars for unlimited texting per month, or t dollars per text message sent or received. If she has m text messages for the month, express the difference in cost between the 'per-text plan and the unlimited plan algebraically.
7. A local cable TV/Internet/phone provider charges new customers \$99 for all three services, per month, for the first year under their "3 for 99" promotion. Joanne normally pays \$54 for her monthly home phone service, \$39 for Internet service, and \$49 for cable television.
- What are her percent savings if she switches to the "3 for 99" plan? Round to the nearest percent.
 - If, after the first year, the flat fee for all three services is \$129, what are her percent savings?
 - Craig usually pays p dollars for phone service, i dollars for Internet service, and c dollars for cable TV service monthly. Represent his savings under the "3 for 99" plan algebraically, as a percent.
8. A cable TV/Internet/phone provider charges Janet \$90 per month for all three services. In addition, Janet's monthly bill for cell phone calls and text messages averages \$77 per month.
- What does Janet pay annually for these services?
 - What is her average cost per day for these services? Round to the nearest cent.
9. Text-Time charges \$25 for a texting plan with 300 text messages included. If the customer goes over the 300 messages, the cost is \$0.10 per message. They have an unlimited plan for \$48 per month.
- If x represents the number of text messages, and $c(x)$ represents the cost of the messages, express $c(x)$ as a piecewise function.
 - Graph the function from part a.
 - What are the coordinates of the cusp in your graph from part b?
 - On the same axes as your graph from part b, graph the function $c(x) = 48$, which represents the cost under the unlimited plan.
 - For what number of text messages are the costs of the two different plans the same?
10. The Fi-Zone cable TV/Internet/phone provider charges \$100 per month for all three services for a new customer's first year. Tobi normally pays p dollars for her monthly home phone service, i dollars for Internet service, and c dollars for cable television. If Fi-Zone estimates a 10% increase in the monthly rate after the first-year special rate, and Tobi's usage remains the same, how much will she save per month in the second year? Express your answer algebraically.
11. Throughout this lesson you have used the greatest integer function $y = [x]$. Use your graphing calculator to view the graph of this function and find the y -coordinate for each of the following x -values.
- 2.3
 - 2.99
 - 3
 - 3.01
 - 3.99
12. The piecewise function below gives the cost $f(x)$ of x text messages per month. Which would be the least expensive plan for a person who had d text messages: a plan in which $c = 31$ or a plan in which $c = 40$? Explain.

$$f(x) = \begin{cases} c & \text{when } x \leq d \\ c + p(x - d) & \text{when } x > d \end{cases}$$

Any sensible family has a budget that lays out how much will be spent for household and other purposes. Without such planning, things would quickly go awry.

Walter Ulbricht, Politician

10-3 Charting A Budget

Objectives

- Create and use a budget check-off matrix.
- Visualize and interpret a budget using a pie chart, a bar graph, a line graph, and a budget line graph.

Common Core

N-VM6, A-REI10, F-IF4, F-IF5, F-IF7a

Key Terms	• budget matrix	• budget check-off matrix	• bar graph
	• matrix	• order of a matrix	• budget line graph
	• row	• pie chart	• sector
	• column	• line graph	• central angle
	• electronic matrix		

Which of the following points is on the line $y = 3x - 1$ and the curve $y = 2x^2 - x - 17$?

CCSS Warm-Up

- a. (4, 60) b. (4, 11) c. (4, 10)

HOW CAN YOU VISUALIZE YOUR BUDGET?

In Chapter 1 you learned to chart stock market data to get a picture of trends, make predictions, examine market strengths and weaknesses, and make decisions on where to invest your money. Those charts served as graphic organizers for understanding the stock market at any given time. Charts that relate to your personal budget can help you understand how and when money is coming in and going out. Perhaps the most common budget chart is the **budget matrix**. A **matrix** is a rectangular array of information. It consists of **rows** and **columns**. A spreadsheet is an **electronic matrix**. A budget matrix can contain numeric entries or information on when certain account deposits and withdrawals are made over a period of time. Examine these samples of two such budget matrices.

Sample A	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Health												
Insurance			✓			✓			✓			✓
Prescriptions (copayments)		✓		✓		✓		✓		✓		✓
Over-the-Counter Meds	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Doctor Visits	✓						✓					
Life Insurance				✓				✓				✓
Health Club Dues									✓			

Sample B	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Health												
Insurance			450			450			450			450
Prescriptions (copayments)		90		90		90		90		90		90
Over-the-Counter Meds	30	30	30	50	50	50	50	50	50	30	30	30
Doctor Visits	200						200					
Life Insurance				300				300				300
Health Club Dues									600			

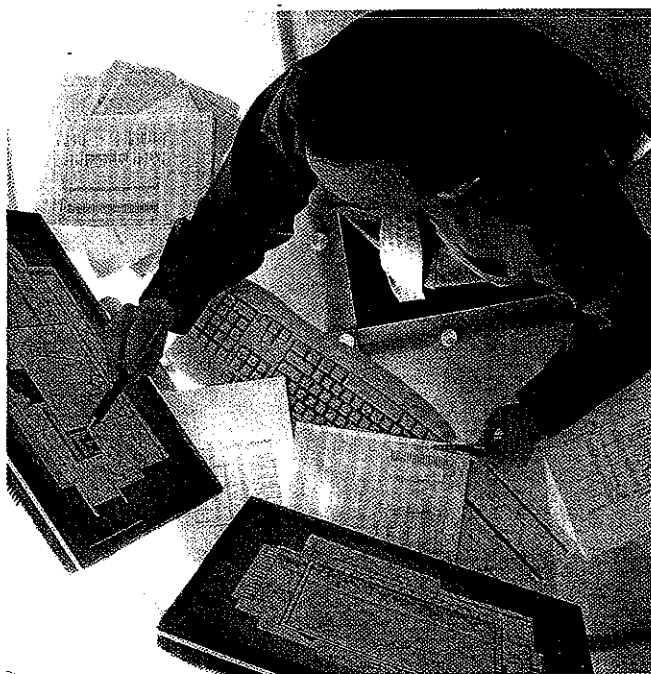
Sample A shows a portion of a yearly budget matrix that relates to health expenses as a monthly **budget check-off matrix**. Notice there are no amounts in this budget. Rather, the check marks indicate at what time of the year the expenses for the categories will occur. Notice December has the most check marks, indicating that the highest health-related expenses *may* be incurred during that month. The user of this budget will have to plan accordingly.

Sample B can be used in addition to the budget shown in Sample A or in place of it. It too shows when expenses will be incurred over the course of a year, but it also gives the amounts of those expenses. Although December had the most number of check marks, notice that September is the month with the highest expenses.

The rectangular format of both budget matrices is exactly the same. Excluding the column and row labels, they each have 6 rows and 12 columns. The number of rows and columns in a matrix is the **order of the matrix**. The order is reported using the form "row \times column" (pronounced "row by column"). Each budget matrix above is a 6 by 12 order matrix.

Budget matrices are not the only ways to visualize financial obligations over time. Other forms of graphical displays are as follows:

- **Pie Chart** A pie chart is a graphic display in the form of a circle divided into pie-shaped segments. A pie chart (or circle graph) is used to present data in percentages.
- **Line Graph** A line graph is used to depict changes over time on a coordinate grid.
- **Bar Graph** A bar graph uses rectangular bars to compare categories of data.
- **Budget Line Graph** This type of graph is used when a consumer is comparing two categories of expenses. It indicates the amount that can be allotted to each expense so that both categories can be afforded within a certain income.



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Skills and Strategies

Here you will learn to use matrices, pie charts, bar graphs, line graphs, and budget line graphs.

EXAMPLE 1

- Create a year-long budget check-off matrix that keeps track of the following household expenses and when they are due.

- Mortgage: monthly
- Utilities: monthly
- Sanitation: quarterly
- Insurance: semiannually
- Internet: semiannually
- Land Line Telephone: monthly
- Cellular Telephone: monthly
- Lawn and Garden: monthly, April–September only
- Snow Removal: November, January, March only
- Food: monthly
- Childcare: every other month beginning with February
- Vet Expenses: semiannually

- **SOLUTION** Based upon the information listed above, construct a 12×12 budget check-off matrix. In addition to these 144 cells (12×12), the top row should include the labels for the months and the first column should include the budget categories as shown.

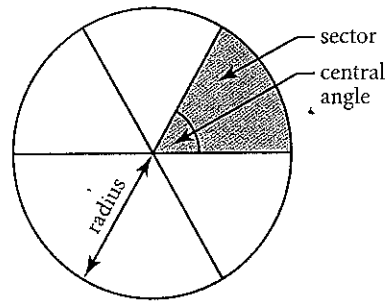
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mortgage	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Utilities	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Sanitation			✓			✓			✓			✓
Insurance						✓						✓
Internet	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Land Line	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Cell	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Lawn/Garden				✓	✓	✓	✓	✓	✓			
Snow	✓		✓								✓	
Food	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Childcare		✓		✓		✓		✓		✓		✓
Vet						✓						✓

■ CHECK YOUR UNDERSTANDING

When setting up a budget, it is important to understand the meaning of the terms used in Example 1 (monthly, quarterly, and semiannually). Suppose that you decide to put two checks in a box when the expense occurred twice in that month. Use a dictionary to determine if that is a *bimonthly expense* or a *semimonthly expense*. Explain the difference between the two terms.

Pie Chart

In order to construct a pie chart, you need to understand a few important characteristics of a circle. The six radii (plural of radius) divide 100% of the area of the circle into six regions called **sectors**. The sectors of the circle are constructed around the center of the circle. The angles formed at the center of the circle are **central angles**. Recall that the sum of the measures of the central angles in a circle is 360 degrees.

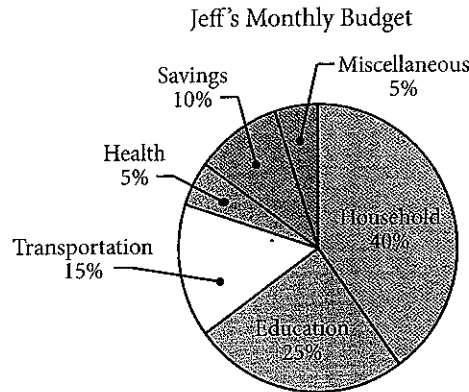


EXAMPLE 2

Jeff budgets his monthly expenses as follows.

- Household: 40%
- Education: 25%
- Transportation: 15%
- Health: 5%
- Savings: 10%
- Miscellaneous: 5%

He used a software program to construct this pie chart to show his expense percentages. How did his category percentages affect the construction of the chart?



SOLUTION The percentage of the total area of each sector corresponds to the percentage of Jeff's money budgeted for each category. Household expenses make up 40% of Jeff's monthly budget, so the central angle forming the household sector is 40% of 360 degrees, or 144 degrees. Once the percentage is determined, the measure of the central angle is calculated by multiplying the decimal equivalent of that percentage by 360 degrees.

Category	%	Measure of Central Angle
Household	40	$0.40 \times 360 = 144$
Education	25	$0.25 \times 360 = 90$
Transportation	15	$0.15 \times 360 = 54$
Health	5	$0.05 \times 360 = 18$
Savings	10	$0.10 \times 360 = 36$
Miscellaneous	5	$0.05 \times 360 = 18$
	100%	360°

The sum of the percentages for this budget equals 100%. The sum of the measures of the central angles of the pie chart equals 360 degrees as shown in the table.

■ CHECK YOUR UNDERSTANDING

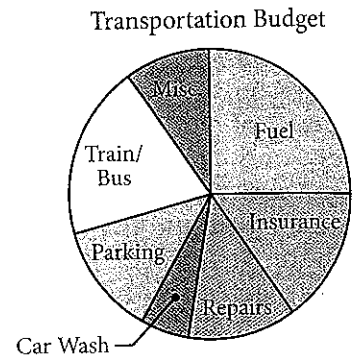
Martha budgets x percent of her monthly income for rent. Express the measure of the central angle for a pie chart sector that represents Martha's rent budget.

EXAMPLE 3

Kate and Paul budget \$800 per month for transportation as shown in the pie chart. What information can you conclude from the pie chart?

SOLUTION Without a protractor to measure the central angles, it is impossible to determine the exact amounts budgeted for each category. However, the following useful information can be seen in the chart.

- Kate and Paul spend the majority of their transportation budget on fuel.
- Repairs and parking appear to have the same amount budgeted.
- Public transportation costs account for the second highest budgeted amount.
- They spend the least amount per month on car washes.
- The amount budgeted for repairs appears to be half the amount budgeted for fuel costs.
- Miscellaneous expenses appear to be half of that budgeted for public transportation.



■ CHECK YOUR UNDERSTANDING

Based upon the information above and knowing that the central angle for the fuel sector is 90 degrees, what information can be determined?

EXAMPLE 4

Construct a bar graph using the information about health-related costs from Sample B on page 496.

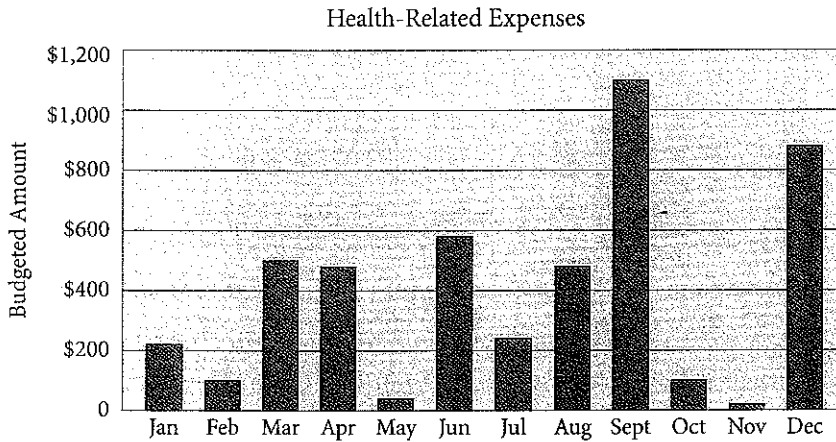
Sample B	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Health												
Insurance			450			450			450			450
Prescriptions (copayments)		90		90		90		90		90		90
Over-the-Counter Meds	30	30	30	50	50	50	50	50	50	30	30	30
Doctor Visits	200						200					
Life Insurance				300				300				300
Health Club Dues									600			

SOLUTION Find the total health-related expenses for each month. The health-related expenses are prescriptions (copayments), over-the-counter medications, and doctor visits.

Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
230	120	480	440	50	590	250	440	1,100	120	30	870

Let the horizontal axis represent months and the vertical axis represent the health-related costs.

Determine the range of the values for the vertical axis. The minimum amount budgeted was \$30 in November. The maximum amount budgeted was \$1,100 in September. Therefore, use a minimum of \$0 and a maximum of \$1,200, with increments of \$200.



■ CHECK YOUR UNDERSTANDING

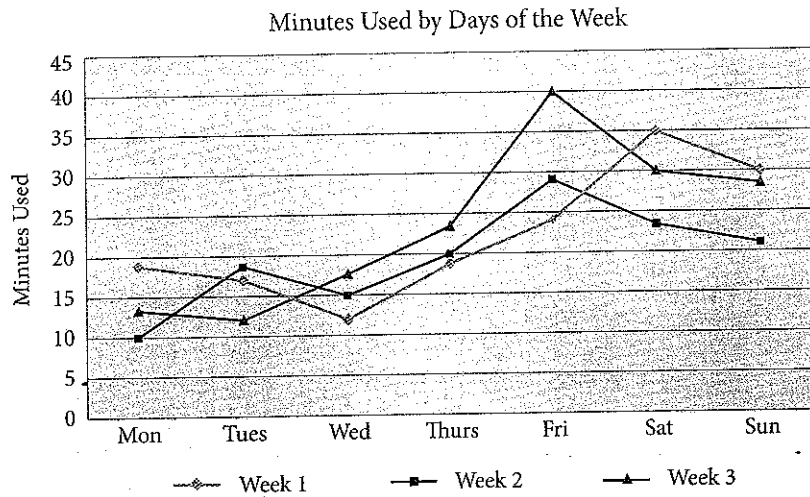
At what amount would a horizontal line be drawn to represent the average monthly budget to the nearest dollar for health-related expenses? In which months was the budgeted amount above average?

EXAMPLE 5

Over the last few months Kate has spent more than her budgeted amount for her cell phone bill. She decided to track her daily usage to see if she should change plans. She went online to her cell phone account and got the usage data below for the last three weeks. Construct a multiple line graph to identify any of Kate's usage trends.

	Week 1	Week 2	Week 3
Mon	18	10	13
Tues	17	19	12
Wed	12	15	18
Thurs	19	20	23
Fri	24	29	40
Sat	35	24	30
Sun	30	22	27

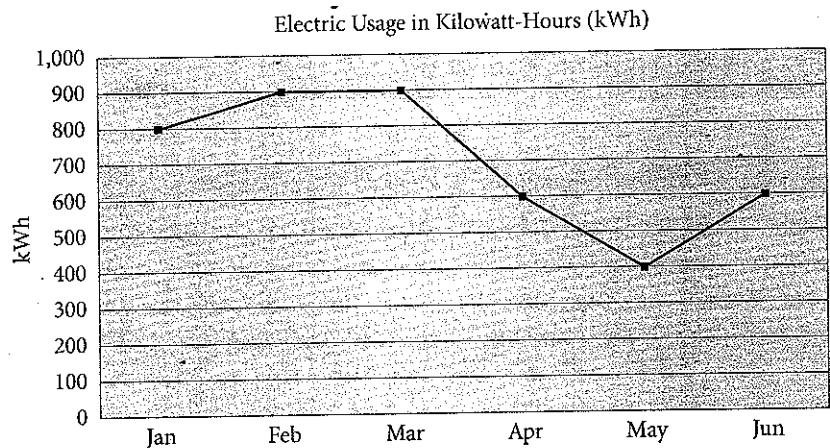
SOLUTION Kate is trying to determine if she needs to adjust her billing plan based upon her cell phone usage. She creates a multiple line graph chart comparing three weeks of phone use. The horizontal axis represents the days of the week and the vertical axis represents the minutes used. Since the maximum number of minutes used was 40 (Friday, Week 3), her vertical scale should be large enough to graph at least that amount. She decided to let the maximum minutes on the vertical scale be 45 with increments of 5.



It appears that Kate does the bulk of her calling on the weekends with the highest number of calls made on a Friday. She should contact her cell phone provider to see if there is a weekend plan available that might reduce her costs and allow her to stay within her budget.

■ CHECK YOUR UNDERSTANDING

Examine the following line graph depicting Claire's electric usage for the first six months of last year.



Her electric company uses this data to calculate her monthly budget for electricity, which is the average of the monthly usage costs. If electricity costs \$0.15714 per kilowatt-hour, how much will her monthly electric budget be? Round your answer to the nearest dollar amount.

EXAMPLE 6

Beth is a coffee lover. In her budget, Beth has a section for coffee. She has budgeted \$90 per month to spend on the coffee she buys in two different locations. At *GasMart*, a cup of coffee costs \$1. At *The Perfect Coffee Company*, a cup of coffee costs \$3. She tries to balance both through a month. Construct a budget line that shows the different combinations of the two types of coffee purchase options which allow her to stay within her budget. Then, suppose she has to decrease her coffee budget by 20%. Identify and graph the new budget line.

SOLUTION If two items, x and y , are budgeted under a single category, their costs C_x and C_y generate the following budget line equation.

$$C_x x + C_y y = B \quad \text{where } B \text{ is the budgeted amount}$$

Let x represent the number of cups of coffee purchased at *GasMart* and y represent the number of cups purchased at *The Perfect Coffee Company*.

Budget line equation $C_x x + C_y y = B$

Substitute $C_x = 1$, $C_y = 3$, and $B = 90$. $x + 3y = 90$

Solve for y . Subtract x from each side. $3y = -x + 90$

Divide each side by 3. $y = -\frac{1}{3}x + 30$

The budget line graph is shown at the right. Three points are shown on the budget line graph.

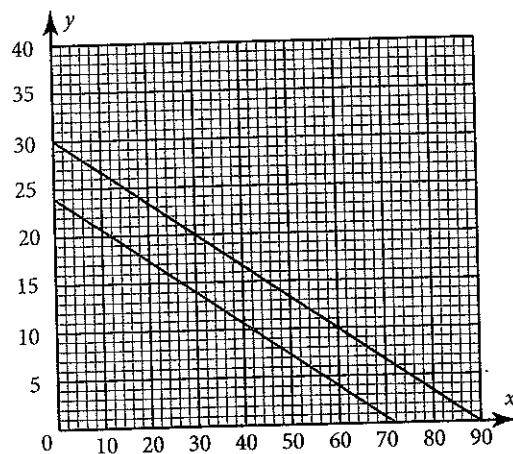
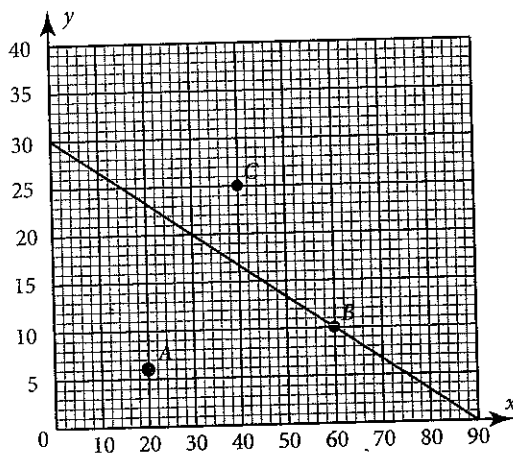
Point A is in the set of all points below the budget line. Any of these points are within the \$90 coffee budget. Therefore, 20 cups of *GasMart* coffee and 6 cups of *The Perfect Coffee Company* coffee cost $20(1) + 6(3)$, or \$38, which is less than the \$90 budgeted.

Point B is in the set of all points that lie on the budget line. If the combination B is chosen (60 *GasMart* cups and 10 *The Perfect Coffee Company* cups), the total cost is exactly the amount budgeted.

Point C is in the set of all points that lie above the budget line. If the combination C is chosen, Beth will be over her budget since $40(1) + 25(3) > 90$.

If Beth has to decrease her coffee budget by 20%, the new budget amount is $0.80(90)$, or \$72. The new budget line equation is $x + 3y = 72$ or $y = -\frac{1}{3}x + 24$. The new budget line (in blue) and the original line are shown in the graph at the right.

Notice that a decrease in the amount budgeted shifts the budget line to the left and decreases the number of combinations that will stay within the budget.



■ CHECK YOUR UNDERSTANDING

Using the coffee budget of \$72, suppose that the price of coffee doubles at *GasMart*. Identify and graph the new budget line equation.

Applications

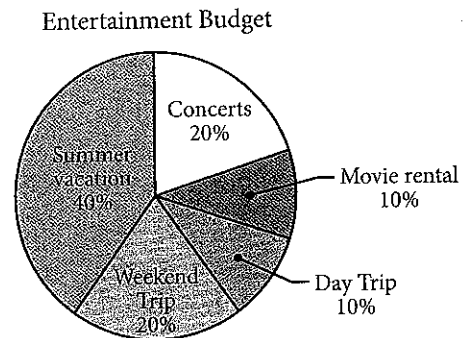
Any sensible family has a budget that lays out how much will be spent for household and other purposes. Without such planning, things would quickly go awry.

Walter Ulbricht, Politician

1. Explain how that quote can be interpreted in light of what you have learned in this lesson.
2. Create a year-long budget check-off matrix to chart the following transportation related expenses: Fuel: monthly; Insurance: quarterly; Servicing: every three months; Car wash: bimonthly; Parking: semi-annually; Public transportation: monthly.
3. Create a year-long budget matrix to chart these expenses: Savings: \$600 bimonthly (starting in January); Retirement account: \$2,000 quarterly; Checking account: \$1,000 semi-monthly; Credit card: \$500 monthly; Life insurance: \$400 semi-annually; Real estate taxes: \$1,300 every four months beginning in April.

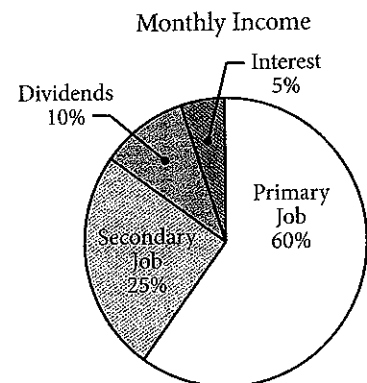
4. Use the pie chart of a monthly entertainment budget.

- a. Suppose \$600 was budgeted. Determine the exact amount for each category.
- b. Suppose \$72 was budgeted for day trips. What would be the total amount budgeted for entertainment?
- c. Suppose \$310 was budgeted for the summer vacation fund. What would be the total amount budgeted for entertainment?

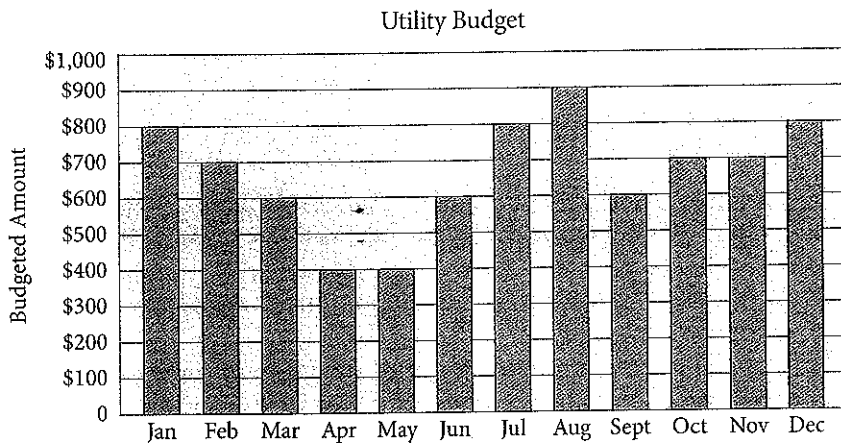


5. Use the pie chart of a person's monthly income.

- a. Suppose the total monthly income is \$5,800. Determine the amounts in each income category.
- b. Suppose the income from the secondary job was \$1,225. What would be the total monthly income?
- c. Suppose the income from interest was \$315. What would be the income from the primary job?



6. Rachel's health-related budget is as follows: The percentage budgeted for health insurance is four times the percentage for health club dues. The percentage budgeted for prescriptions is equal to one-fourth the percentage budgeted for health insurance. The percentage budgeted for doctor visits is twice the percentage budgeted for prescriptions.
- What is the percentage for health insurance?
 - What are the percentages budgeted for all categories?
 - If \$900 is budgeted for health-related expenses, how much is budgeted for health insurance?
7. Construct a pie chart that shows the following transportation-related expenses: Fuel: \$240; Insurance: \$80; Public transportation: \$200; Parking garage: \$120; Repairs: \$160.
8. Examine the following bar graph that shows budgeted monthly utility expenses for a one-year period.

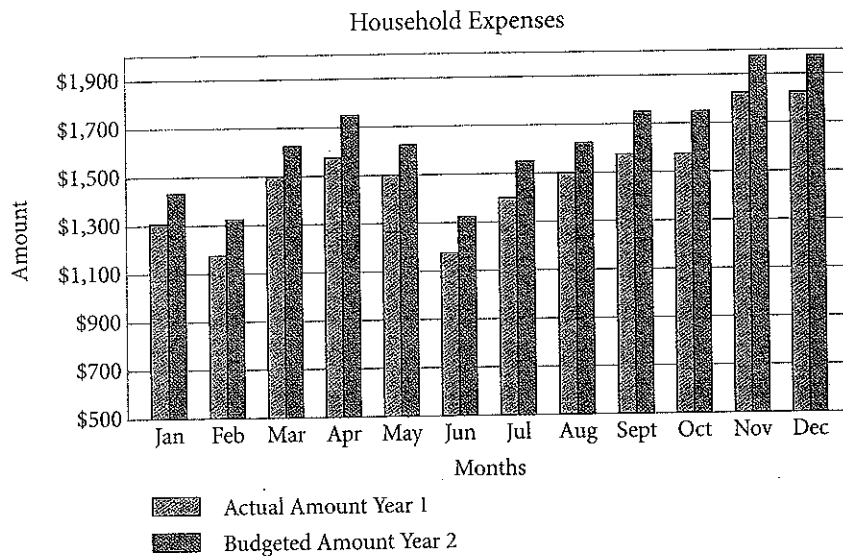


- In which months was the same amount budgeted?
 - What is the total annual amount budgeted for utilities?
 - What percent of the total yearly amount was budgeted for the warm-weather months of June–September?
 - Between which months was there a 100% increase in the amounts budgeted for utilities?
 - At the end of December in the year shown, the homeowners replaced their furnace with a more energy-efficient one. They were told that they could decrease their utility budget for the upcoming month of January by 20% from the previous January amount. How much will they budget for utilities in January?
9. Construct a bar graph for Jason's transportation budget expenses.

400	550	400	650	400	350	200	300	450	500	650	500
Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec

10. Construct a bar graph for the September budget category "Personal Items" using the following amounts: Haircut: \$20, Clothing purchases: \$60, Books: \$20, Newspapers/Magazines: \$65, Online subscriptions: \$30, Gifts: \$80, Donations \$40, Other: \$50.

11. Aida created the following double bar graph. It illustrates the actual amount spent this year for household expenses per month and the budgeted amounts for the following year. The following year's amounts reflect a 5% increase over the cost of living adjustment (COLA).



For June, the actual amount is \$1,200 and the budgeted amount for Year 2 is \$1,320.

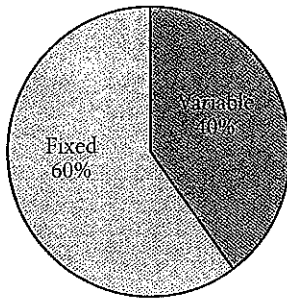
- What was the cost of living adjustment that Aida used to get the budgeted amount?
 - Determine the budgeted amount for January of Year 2.
 - The August actual amount was the exact amount Aida had budgeted for. This actual amount reflected an 8% increase over the previous year's actual amount for August. What was the previous year's actual amount for August rounded to the nearest dollar?
12. Mark works at two jobs. His primary job is a commission-paid job. Therefore, his monthly income from this job varies. His secondary job has a fixed monthly income. His quarterly dividend checks and interest income have varied but not by a large amount. He used actual amounts from the previous year to set up this budget for the upcoming year.

	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Primary Job	7,000	8,500	9,000	8,000	8,000	8,500	9,000	8,000	7,000	6,500	7,000	7,000
Secondary Job	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000
Interest			380			390			400			410
Dividends			150			150			150			150
Other						3,000						3,000

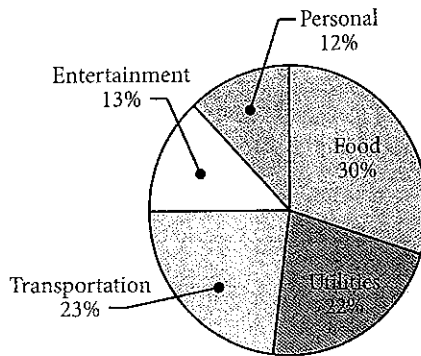
- What is his budgeted income for each month?
- Construct a line graph to depict this budget.
- Determine the average monthly income. Round your answer to the nearest dollar.
- Draw a horizontal line on your graph representing that amount. What months fell below the average?

13. Rich budgets \$2,500 in expenses for the month of January. He constructed the two graphs below.

January Budget



January Budget - Variable Expenses



Using prior year's data and the budget he has created, he has budgeted the transportation category for the year as follows.

Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Use charts	+10%	+5%	+3%	+5%	+8%	+2%	-5%	-3%	+5%	+8%	+10%

Each of the monthly percentages is an increase or decrease from the January amount.

- How much did Rich budget for fixed and variable expenses in January?
 - How much did he budget for transportation?
 - Determine the monthly budgeted amounts for transportation based upon your answer to part b.
 - Construct a line graph to chart the changes in Rich's transportation budget over time.
14. Under his household expense budget category, Mark has allocated \$60 per month for pet food. He can purchase wet food in a can for \$1.50 per can or dry food in a bag for \$3 per bag.
- Determine a budget line equation for this situation.
 - Graph the budget line that will depict the different combinations of cans and bags that Mark can purchase while still remaining within his budget.
 - Name a combination of bags and cans that will allow him to meet his budget exactly.
 - Name a combination of bags and cans that will keep him under budget.
 - Name a combination of bags and cans that will cause him to be over budget.
15. A consumer budgets \$480 per month for transportation. She has determined that the cost of a round-trip train ride is \$4 and the cost of each round-trip car ride (factoring in gas, oil, etc.) is \$3.
- Write a budget line equation for this situation.
 - Construct the budget line graph that models this situation.
 - What do the points on the budget line represent?
 - What do points below the budget line represent?
 - Suppose that the budgeted amount increases to \$516. Construct the new budget line and the old budget line on the same axes.
 - What does the region in between the two lines represent?

10-4 Cash Flow and Budgeting

Objectives

- Develop and interpret a cash flow chart.
- Develop and interpret a frequency budget plan.
- Develop and interpret a year-long expense budget plan.

Common Core

A-SSE1, F-BF1

Key

Terms

- cash flow analysis
- cash flow
- pro-rate
- envelope accounting system
- frequency budget plan
- year-long expense budget plan
- net worth
- assets
- liabilities
- debt reduction plan
- debt-to-income ratio

Express algebraically:

"The average of x , y , and $2w$ is k more than 15."

CCSS Warm-Up

HOW DO YOU PLAN FOR EXPENSES, REDUCE DEBT, AND GROW SAVINGS?

A budget is more than just a matrix of numbers that charts your income and expenditures. It is a well-thought-out plan that is the result of careful examination of your financial goals and obligations. Budgets allow you to plan for future spending and saving. They give you control over your financial situation and allow you to make financially sound modifications to your lifestyle based upon comparing what was planned and what actually occurred.

Budgets can also be motivators. You can celebrate when you accomplish your goals, and you can reflect and adjust when you find that you were not able to attain what you had planned for. A budget can be the product of both financial and nonfinancial goals. For example, you might have a goal of taking a vacation in the upcoming year, or eating healthier, or improving your public speaking. Each of these goals could contribute to the formation of your budget. Perhaps the most common financial goals that people make have to do with growing their savings and reducing their debt. Once you have selected your goals, it is then important to take a close look at your current income and spending habits. This is a **cash flow analysis**. A cash flow analysis is not a budget. It has nothing to do with a financial plan nor does it incorporate goals. Rather, it is a detailing of how money comes in and how money goes out over a fixed period of time. By carefully monitoring your **cash flow**, you can get a better sense of the possibility of meeting your goals.

There are many cash flow analyzers and budget planners on the Internet. You can find many sites with a variety of templates that you can adjust to meet your own specific needs. Cash flow analysis will help you determine whether or not you are "living within your means." If your monthly cash flow is a positive number, this implies that your income is more than what you spend. You can decide whether to carry the extra over to the following month, to increase your savings, or to increase the amount to pay off debt. If your monthly cash flow is negative, it means you are spending more money than you are making. You will have to readjust your plan so that you do not end up borrowing money at the end of each month.

Skills and Strategies

Here you will learn how to use cash flow analysis to set up a budget.

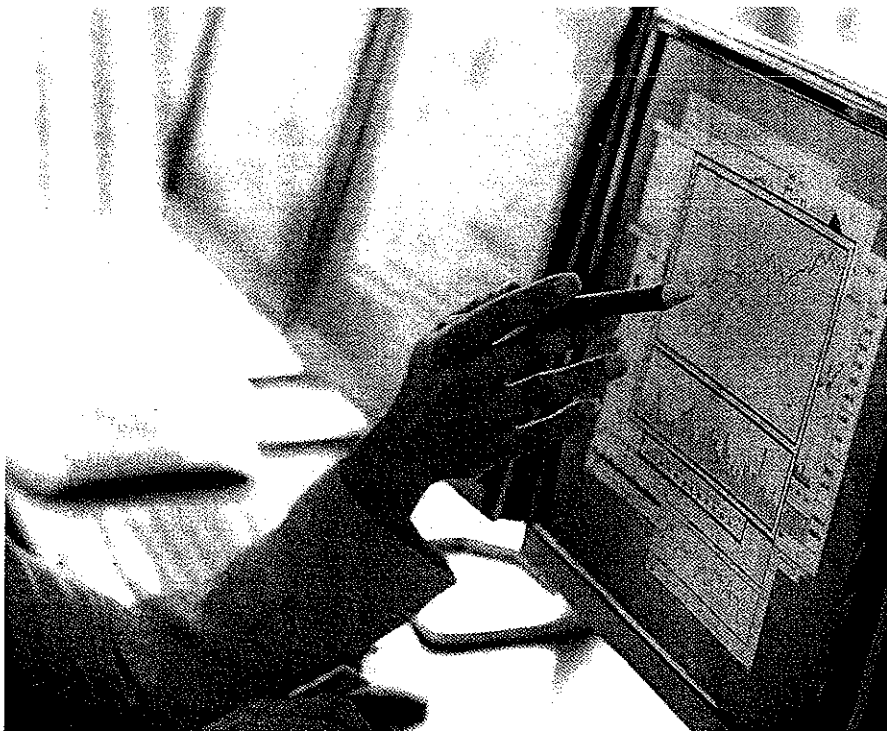
EXAMPLE 1

Dave and Joan want to chart their monthly cash flow. Create a spreadsheet that will help them keep track of their income and expenses for the month.

SOLUTION Dave and Joan should begin by collecting monthly amounts for the following categories.

- **Income** Make a list of all after-tax income. This includes primary and secondary jobs, unemployment benefits, Social Security, interest, dividends, child support, alimony, annuity income, gifts, etc.
- **Fixed Expenses** These expenses are the same from month to month. For example, mortgage/rent, loan payments, insurance premiums, and so on.
- **Variable Expenses** These expenses change each month. Your food and utility expenses are variable.
- **Savings** Financial advisors suggest that savings be considered a fixed expense.

The cash flow analysis spreadsheet shown on the next page asks the user to input after-tax income in cells B2–B4. Dave and Joan's combined after-tax income is \$5,600 from their primary jobs. They both make extra money helping out at a family-owned business on Saturdays. That income is entered in cell B3. The total income is in cell B5.



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	A	B	C	D	E
1	Income				
2	Primary Employment	5,600			
3	Secondary Employment	500			
4	Other Income				
5	Total Income	6,100			
6	Fixed Expenses			Non-Monthly Expenses (per year)	
7	Rent/Mortgage	2,400		Medical/Dental	600
8	Car Loan Payment	200		Auto Related	700
9	Education Loan Payment	150		Home Related	500
10	Personal Loan Payment	80		Life Insurance	600
11	Health Insurance Premium	50		Tuition	3,000
12	Life Insurance Premium			Vacation	1,200
13	Car Insurance Premium	60		Gifts	400
14	Homeowner's/Renter's Insurance	50		Contributions	400
15	Cable TV			Repairs	600
16	Total Fixed Expenses	2,990		Taxes	4,000
17				Other	
18	Variable Expenses			Total Non-Monthly Expenses (per year)	12,000
19	Groceries (Food)	800			
20	Dining Out	150		Total Non-Monthly Expenses (per month)	1,000
21	Fuel (Car)	160			
22	Cell Phone	120		Total Expenses	5,990
23	Land Line Phone			Monthly Cash Flow	110
24	Electricity	80			
25	Water	30			
26	Sewer				
27	Sanitation				
28	Medical				
29	Entertainment	200			
30	Savings	300			
31	Debt Reduction	160			
32	Other				
33	Total Variable Expenses	2,000			

- The fixed and variable expenses categories are monthly bills. Notice that Dave and Joan left some of those cells empty. They only have cellular phones, and they do not pay any fees for sewer usage. Their life insurance, sanitation, and medical expenses are not billed monthly so those expenses are accounted for in another category.
- Not all bills are monthly. Some, like insurance, tuition, vacation, medical/dental may be bimonthly, quarterly, semi-annually, or at varying intervals. It is still possible to prepare for those bills by examining their impact if spread evenly over the 12 months of the year.

• The cash flow spreadsheet category Non-Monthly Expenses (per year)
• asks the user to input the accumulated yearly amount for any expense
• that does not occur on a monthly basis. For example, Dave and Joan pay
• their life insurance bill of \$150 every three months. That is four times a
• year for a year-long total of \$600. Six hundred dollars has been entered
• in cell E10.

• The total year-long amount appears in cell E18. The cash flow spread-
• sheet divides that amount by 12, and enters the monthly amount in
• cell E20.

• In order to be prepared for these non-monthly expenses, it is important
• to **pro-rate** them, that is, to divide them proportionately as if they were
• monthly expenses. Notice that Dave and Joan have a total of \$12,000
• in non-monthly expenses for the year. They need to account for \$1,000
• each month in order to be able to meet those expenses when the bills
• arrive. This monthly amount can be deposited into a special savings
• account and drawn upon when needed at various intervals in the year.

• Cell D23 indicates that Dave and Joan have a positive cash flow of
• \$110. They can carry that money over to the following month for
• emergency situations or they can choose to increase their savings or
• decrease their debt by that amount.

■ CHECK YOUR UNDERSTANDING

Suppose that the cash flow had been $-\$160$. What advice might you give to Dave and Joan?

EXAMPLE 2

• The Consumer Credit Counseling Service suggests that transportation
• expenses be between 6–20% of your budget and savings be between
• 5–9%. Using Dave and Joan's cash flow analysis, determine whether
• they remain within the guidelines for these categories.

• **SOLUTION** Dave and Joan allocated \$200 monthly for their car loan
• payment, \$50 for their car insurance premium, and \$160 for gasoline.
• They had \$700 worth of auto-related expenses spread over the course of
• the year. Rounding up, this is approximately \$59 per month. The total
• transportation expenses budgeted is \$469. Since their total monthly
• income is \$6,100, transportation accounts for approximately 7.7%
• ($469 \div 6,100$) of their budget. This is slightly over the suggested amount.

• Their budgeted amount of \$300 per month for savings is about 4.9%
• ($300 \div 6,100$) of their income, which is slightly lower than the sug-
• gested percentage.

■ CHECK YOUR UNDERSTANDING

Dave and Joan want to include a section in their cash flow spreadsheet that will calculate the monthly percentage allocated to certain categories suggested by the Consumer Credit Counseling Service. Write the spreadsheet formula that will calculate the transportation percentage for the month.

Create, Use, and Modify a Budget

Once the cash flow analysis has been completed, it is time to convert what you have learned into a working budget. A budget is a personalized plan. A budget that works for one person may not work for another. A budget can be a simple system. The **envelope accounting system** is a way to manage your income with real dollars rather than with formulas and numbers in a matrix. Envelopes are set up to hold the allocated amount for weekly or monthly budget categories. Suppose you decide to budget \$50 per week for dining out. This could include breakfast, lunch, dinner, or even snacks and coffee. Any time you pay for any item in this category, you take the money from the dining-out envelope. Once the money runs out, you have to wait until the next envelope allocation period comes around (usually on payday). If there is money remaining in the envelope at the end of a cycle, that money can be transferred to another envelope where funds are needed more. This system can work for people who have very few budgeting categories. College students might find this advantageous.

As financial responsibilities and expenses grow, so too does the sophistication of the budget. Examples 3 and 4 show two spreadsheet budgets using Dave and Joan's cash flow worksheet from Example 1. Example 3 shows a household budget in terms of the frequency that payments or credits are made over the course of a year, called a **frequency budget plan**. Example 4 shows a **year-long expense budget plan** in which entries are made under each of the months of the year.

Whichever budget structure you decide to use, it is important to review your budget periodically. Make alterations where necessary. Shift allocations to increase savings and reduce debt. Always pay your bills on time to avoid late charges. Keep accurate records. Set achievable goals and review them regularly.

EXAMPLE 3

- Create a frequency budget plan for Dave and Joan using their cash flow analysis from Example 1.

• **SOLUTION** On the next page is a frequency budget plan using the data from Dave and Joan's cash flow analysis. A budget need not be electronic. Everything a spreadsheet can be programmed to do can also be done by hand (with or without a calculator). The frequency budget shown is a template that can be filled in by hand. In the applications, you will be asked to convert it into a spreadsheet.

• The frequency budget plan is built on the premise that most expenses and income deposits occur at predetermined intervals over the course of the year. Income and the frequency of that income are recorded at the top of the budget worksheet. The annual income is calculated by finding the product of the interval income and the frequency. The expenses are categorized by the frequency.

■ CHECK YOUR UNDERSTANDING

The frequency budget, shown on the facing page, states that Dave and Joan have an annual surplus of \$1,284. How does this relate to the monthly positive cash flow that was computed in Example 1?

Frequency Budget Plan

After-Tax Income Categories	Income Amounts	Frequency	Annual Amount
Primary Employment	2,800	24	67,200
Secondary Employment	500	12	6,000
Interest			
Dividends			
Other Income			
Total Income			73,200
Weekly Expenses	Expense Amounts		
Food	185	52	9,620
Personal Transportation	37	52	1,924
Public Transportation		52	0
Household		52	0
Childcare		52	0
Dining Out	35	52	1,820
Entertainment	46	52	2,392
Other		52	0
Total Weekly Expenses			15,756
Monthly Expenses			
Mortgage	2,400	12	28,800
Utilities	110	12	1,320
Land Line/Cellular Telephone	120	12	1,440
Car Loan	200	12	2,400
Education Loan	150	12	1,800
Personal Loan	80	12	960
Car Insurance	60	12	720
Homeowner's Insurance	50	12	600
Savings	300	12	3,600
Debt Reduction	160	12	1,920
Other	50	12	600
Total Monthly Expenses			44,160
Other Frequency Expenses			
Medical/Dental	600	1	600
Auto-Related	700	1	700
Home-Related	250	2	500
Life Insurance	150	4	600
Tuition	1,500	2	3,000
Vacation	1,200	1	1,200
Gifts	200	2	400
Contributions	400	1	400
Repairs	600	1	600
Taxes	2,000	2	4,000
Other			
Total Other Frequency Expenses			12,000
Total Expenses			71,916
Annual Surplus or Deficit			1,284

EXAMPLE 4

Construct a year-long expense budget spreadsheet using the cash flow data from Dave and Joan.

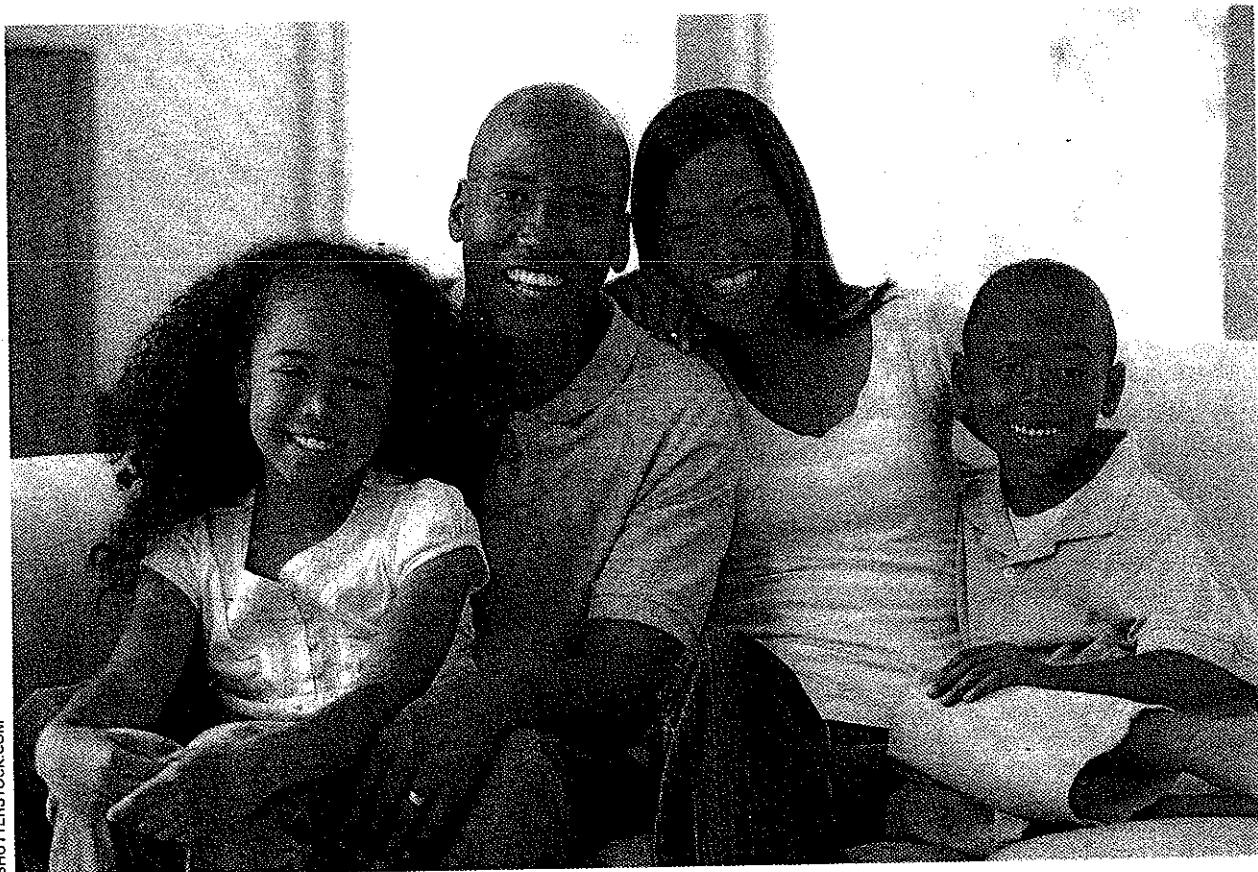
SOLUTION A year-long expense budget spreadsheet lays out the year's income and expenses by month. It gives the user a snapshot of financial expectations throughout the year. It can be modified as the year progresses.

The sample year-long budget shown on the next page is a month-by-month accounting of money coming in and money going out. It should be used in conjunction with a cash flow analysis. Every few months, the budget should be reevaluated and adjusted based upon any changes in the financial situation. Dave and Joan's year-long budget indicates that there are some months in which the expenses will be higher than others and they will have to plan accordingly.

■ CHECK YOUR UNDERSTANDING

Use the spreadsheet to create row 43 in which the totals for each month will be calculated. What formula would be used for January? What entries will appear for each of the months in this row if the same formula is applied to the remaining months?

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	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Income	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2	Primary Employment	5,600	5,600	5,600	5,600	5,600	5,600	5,600	5,600	5,600	5,600	5,600	5,600
3	Secondary Employment	500	500	500	500	500	500	500	500	500	500	500	500
4	Other Income												
5	Total Income	6,100	6,100	6,100	6,100	6,100	6,100	6,100	6,100	6,100	6,100	6,100	6,100
6													
7	Fixed Expenses												
8	Rent/Mortgage	2,400	2,400	2,400	2,400	2,400	2,400	2,400	2,400	2,400	2,400	2,400	2,400
9	Car Loan Payment	200	200	200	200	200	200	200	200	200	200	200	200
10	Education Loan Payment	150	150	150	150	150	150	150	150	150	150	150	150
11	Personal Loan Payment	80	80	80	80	80	80	80	80	80	80	80	80
12	Health Insurance Premium	50	50	50	50	50	50	50	50	50	50	50	50
13	Life Insurance Premium												
14	Car Insurance Premium	60	60	60	60	60	60	60	60	60	60	60	60
15	Homeowner's Insurance	50	50	50	50	50	50	50	50	50	50	50	50
16	Cable TV												
17	Life Insurance			150			150			150			150
18	Tuition	1,500							1,500				
19	Taxes		2,000							2,000			
20													
21	Variable Expenses												
22	Groceries (Food)	800	800	800	800	800	800	800	800	800	800	800	800
23	Dining Out	150	150	150	150	150	150	150	150	150	150	150	150
24	Fuel (Car)	160	160	160	160	160	160	160	160	160	160	160	160
25	Cell Phone	120	120	120	120	120	120	120	120	120	120	120	120
26	Land Line Phone												
27	Electricity	80	80	80	80	80	80	80	80	80	80	80	80
28	Water	30	30	30	30	30	30	30	30	30	30	30	30
29	Sewer												
30	Sanitation												
31	Medical												
32	Medical/Dental												600
33	Auto-related										700		
34	Home-related				250				250				
35	Vacation							1,200					
36	Gifts		200									200	
37	Contributions					400							
38	Repairs										600		
39	Entertainment	200	200	200	200	200	200	200	200	200	200	200	200
40	Savings	300	300	300	300	300	300	300	300	300	300	300	300
41	Debt Reduction	160	160	160	160	160	160	160	160	160	160	160	160
42	Other												

Net Worth

Budgeting and cash flow help you understand how to manage your money so that there is a financially responsible balance between the money coming in and the money going out.

In addition to this type of planning, it is also a good idea to ask yourself the question "Where do I stand financially today?" To do so, you need to calculate your net worth. **Net worth** is the difference between your **assets** (what you own) and your **liabilities** (what you owe). The result can be a positive or a negative number.

You should calculate your net worth at regular intervals of perhaps once or twice a year, to determine if there have been changes in your financial status.

EXAMPLE 5

Liam Brown is single, in his mid-twenties, and owns a condo in a big city. He has calculated the following assets and liabilities.

Assets

- Current value of condo: \$580,000
- Current value of car (as listed in Kelley Blue Book): \$17,000
- Balance in checking account: \$980
- Combined balance in all savings accounts: \$22,500
- Current balance in retirement account: \$24,800
- Current value of computer: \$2,900
- Current value of collector bass guitar: \$6,700
- Current value of stocks/bonds: \$18,300

Liabilities

- Remaining balance owed on home mortgage: \$380,000
 - Remaining balance owed on student loans: \$51,000
 - Combined credit card debt: \$1,600
- Calculate Liam's net worth. Last year at this time, he calculated his net worth as \$205,780. Compare both values. What do the changes mean?

SOLUTION Net worth is the difference between assets and liabilities.

• Find the sum of the assets.

$$580,000 + 17,000 + 980 + 22,500 + 24,800 + 2,900 + 6,700 + 18,300 = 673,180$$

• Find the sum of the liabilities. $380,000 + 51,000 + 1,600 = 432,600$

• Find the difference. $673,180 - 432,600 = 240,580$

• Liam's current net worth is \$240,580.

• This is a \$34,800 increase in net worth from the previous year. This increase represents an improving trend in Liam's financial well-being.

■ CHECK YOUR UNDERSTANDING

What can Liam do to continue his improving net worth trend?

Debt Reduction Planning

One of the most important things that Liam can do is to create a **debt reduction plan**. A thoughtful debt reduction plan requires an honest accounting of an individual's net worth as well as a calculation of a personal **debt-to-income ratio**. This ratio, expressed as a percent, offers some stark reality as to where you stand financially with the amount of debt presently being carried.

A ratio of 15% and under is recommended. A ratio of 20% and higher should alert you to the fact that you need to put a debt reduction and spending plan in place.

Debt reduction advice can be found in book, video, and online formats. It is often suggested that you can improve your debt-to-income ratio by setting up a debt reduction plan that contains some or all of the following.

- Lower debts that have the highest interest rates.
- Pay more than the minimum amount whenever possible.
- Look for ways to cut costs daily.
- Use an online debt management calculator.
- Face your debt. Make a list of what you owe and keep that handy to discourage you from incurring more debt.
- Slow down or eliminate your credit card spending.

After doing some research, you should develop a plan that is best for you and above all, do everything you can to stick with that plan.

EXAMPLE 6

Tome's monthly liabilities and assets are as shown in the table.

Monthly Liabilities (Debt)		Monthly Pre-Tax Assets (Income)	
Mortgage Payment	\$2,300	Gross Salary	\$7,800
Student Loan Payment	\$ 750	Stock Dividends	\$ 380
Minimum Credit Card Payment	\$ 200	Interest	\$ 120
Car Loan Payment	\$ 150		

Find Tome's debt-to-income ratio. Express that ratio as a percent.

SOLUTION

Find the sum of Tome's assets. $7,800 + 380 + 120 = 8,300$

Find the sum of Tome's liabilities. $2,300 + 750 + 200 + 150 = 3,400$

$$\text{Debt-to-income ratio} = \frac{\text{Debt}}{\text{Pre-tax income}} = \frac{3,400}{8,300} \approx 0.405$$

Tome's debt-to-income ratio is approximately 40.5%, which is very high.

■ CHECK YOUR UNDERSTANDING

Tome anticipates that next year, his car and student loans will have been paid off and he will have received a 10% salary increase. If everything else remains the same, calculate that debt-to-income ratio.

Applications

Budget: a mathematical confirmation of your suspicions.

A. A. Latimer

1. Explain how the quote can be interpreted in light of what you learned.

Use the table below for Exercises 2 and 3.

	July	Aug	Sept	Oct	Nov	Dec	Average
Groceries (Food)	740	800	650	820	820	880	a.
Dining Out	120	150	300	80	100	150	b.
Fuel (car)	200	240	320	300	280	220	c.
Cell Phone	104	108	126	140	120	104	d.
Land Line Phone	80	90	60	80	100	88	e.
Electricity	140	160	120	90	140	160	f.
Water	52	58	62	48	48	62	g.

2. The Larsons use the average of six months as their budget starting point in each category. Find each average.
3. The Larsons (from Exercise 2) decided they would not use the September amounts, when they were on vacation, nor the December amounts, when cousins stayed with them. Assume the chart is a spreadsheet with row 1 as the month labels and column A as the expense categories.
 - a. Write a spreadsheet formula to calculate the adjusted average for food.
 - b. What is the adjusted average for food?
 - c. Write a spreadsheet formula to calculate the adjusted average for electricity.
 - d. What is adjusted average for electricity?
 - e. Why do you think that the adjusted average is not that far off from the six-month average in each category?

Assets

Current value of home	\$422,000
Current value of car (<i>Kelley Blue Book</i>)	\$22,000
Balance in checking account	\$2,380
Balance of savings accounts	\$140,500
Balance of retirement account	\$250,000
Value of computer	\$1,800
Value of stocks/bonds	\$67,000

Liabilities

Balance owed on home mortgage	\$120,000
Balance owed on home equity loan	\$21,000
Combined credit card debt	\$940

4. Bob Forrester is retired and owns a home. He has these assets and liabilities.
 - a. Calculate Bob's net worth.
 - b. Two years ago, Bob's net worth was \$650,000. Last year, his net worth was \$740,500. What is the approximate percent of increase or decrease between two years ago and last year?
 - c. What is the approximate percent of increase or decrease between last year and this year?
 - d. Compare the values. What do the changes imply?

Use the budget information for Laura Shannon for Exercises 5–11.

5. The Consumer Credit Counseling Service suggests that the monthly food budget be between 15–30% of income.
 - a. What is Laura’s total monthly food bill including dining out?
 - b. What percent of her income is spent on food?
 - c. Is Laura below, in, or above the recommended interval?
6. Examine Laura’s non-monthly expenses.
 - a. Which month has the greatest expenses?
 - b. How might Laura prepare for those expenses?
7. Use a cash flow template to construct a cash flow plan for Laura. What is her monthly cash flow?
8. Create a frequency budget for Laura as on page 513. Although her food, fuel, dining out, and entertainment expenses were listed monthly for the cash flow, they should be considered weekly expenses here. Round the weekly amount up to the nearest dollar. Combine electricity, water, and sanitation under the utilities category. Any categories not mentioned belong in the other section. According to this frequency budget plan, what is Laura’s surplus or deficit for the year?
9. Create a spreadsheet for the frequency budget in Exercise 8.
10. Create a year-long budget for Laura as on page 515.
11. Add row 43 to your matrix from Exercise 10. Calculate each month’s total expenses.
 - a. Draw a line graph to chart Laura’s monthly budgeted expenses. Include horizontal lines to indicate the average monthly expense and the monthly income.
 - b. What advice would you give to Laura based upon the graph you have constructed?
12. Marina’s monthly liabilities and assets are as shown in the table.
 - a. Find Marina’s debt-to-income ratio. Express that ratio as a percent.
 - b. How would you categorize her debt-to-income ratio?

Laura’s Financial Report			
Income			
Teacher, monthly after-tax income: \$5,000			
Tutor, monthly after-tax income: \$1,200			
Monthly Expenses			
Rent	\$2,200	Groceries	\$600
Car loan	\$180	Personal loan	\$100
Electricity	\$80	Land line phone	\$60
Sanitation	\$50	Auto insurance	\$70
Cable/Internet	\$40	Savings	\$400
Dining out	\$200	College loan	\$250
Gasoline	\$200	Cell phone	\$80
Water	\$40	Medical insurance	\$60
Renter’s insurance	\$30	Entertainment	\$250
Debt reduction	\$200		
Non-Monthly Expenses			
Medical: \$250 in April, \$250 in September			
Auto-related: \$400 in October			
Home-related: \$250 in February, \$250 in November			
Life insurance: \$110 in April, August, December			
Tuition: \$2,000 in May (summer) and August (fall)			
Vacation: \$800 in July			
Gifts: \$250 in March and December.			
Contributions: \$10 each week of the year			
Repairs: \$280 in October			
Taxes: \$1,500 in January and September			

Monthly Liabilities (Debt)		Monthly Pre-Tax Assets (Income)	
Rent	\$1,400	Gross Salary	\$8,900
Student Loan Payment	\$ 350	Stock Dividends	\$ 350
Minimum Credit Card Payment	\$ 140	Interest	\$ 190
Car Loan Payment	\$ 130		
Graduate School Loan	\$ 600		